

# Covariant Description of Flavor Conversion in the LHC Era

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## Abstract

A simple covariant formalism to describe flavor and CP violation in the left-handed quark sector in a model independent way is provided. The introduction of a covariant basis, which makes the standard model approximate symmetry structure manifest, leads to a physical and transparent picture of flavor conversion processes. Our method is particularly useful to derive robust bounds on models with arbitrary mechanisms of alignment. Known constraints on flavor violation in the  $K$  and  $D$  systems are reproduced in a straightforward manner. Assumptions-free limits, based on top flavor violation at the LHC, are then obtained. In the absence of signal, with  $100 \text{ fb}^{-1}$  of data, the LHC will exclude weakly coupled (strongly coupled) new physics up to a scale of 0.6 TeV (7.6 TeV), while at present no general constraint can be set related to  $\Delta t = 1$  processes. LHC data will constrain  $\Delta F = 2$  contributions via same-sign tops signal, with a model independent exclusion region of 0.08 TeV (1.0 TeV). However, in this case, stronger bounds are found from the study of CP violation in  $D - \bar{D}$  mixing with a scale of 0.57 TeV (7.2 TeV). In addition, we apply our analysis to models of supersymmetry and warped extra dimension. The minimal flavor violation framework is also discussed, where the formalism allows to distinguish between the linear and generic non-linear limits within this class of models.

## 1 Introduction

The standard model (SM) has a unique way of incorporating CP violation (CPV) and suppressing flavor changing neutral currents (FCNCs). In fact, the way the SM flavor symmetry is broken to allow flavor conversion is quite intriguing. It can be described in the language of collective breaking [1, 2], a term commonly used in the Little Higgs literature (see *e.g.* [3] and refs. therein). Inter-generation transitions require the presence of non-universal Yukawa couplings for both up and down quarks, a non-vanishing weak coupling and a non-trivial CKM matrix. The lightness of the first two generation masses and the approximate alignment between the Yukawa matrices further suppress FCNC transitions involving the first two generation. This is manifest in particular in processes which are characterized by hard GIM, such as ones involving CPV. Inclusive third generation processes are further simplified due to the presence of an approximate residual  $U(1)_Q$  symmetry [4], only broken by the mass differences of the light quarks.

New types of microscopic dynamics with a different flavor breaking machinery typically give rise to deviation from the SM approximated selection rules, and hence can be distinctively distinguished from the SM. Till today no deviation from the SM predictions related to quark flavor

violation has been observed<sup>1</sup>. This probably implies that new physics (NP) searches should focus on SM extensions which, if not flavor blind, share some of the structure and properties described above.

Regarding the first two generations, models which do not include some sort of degeneracies or flavor alignment (that is, when NP contributions are diagonal in the quark mass basis) are bounded to a high energy scale. Moreover, contributions involving only quark doublets cannot be simultaneously aligned with both the down and the up mass bases, hence even alignment theories are constrained by measurements. However, the hierarchy problem is not triggered by the light quarks, but rather by the large top Yukawa, where almost any natural NP model consists of an extended top sector. In addition, within the SM, the top dominates the CP violating transitions, and dials the amount of custodial symmetry breaking. Ironically, the top sector is the least experimentally explored, and at present model independent bounds on its flavor violating couplings are rather poor.

In this work, we elaborate on a basis independent formalism for studying flavor constraints in the quark sector, that was recently introduced by us in [4] (see also [6] for related work about algebraic flavor invariants). Apart from yielding a simple, symmetry driven, manner to understand the SM way of breaking flavor and CP, it also provides a straightforward method to study generic forms of NP flavor violation and derive model independent bounds (focusing on the left-handed quark sector and assuming  $SU(2)_L$ -invariant NP contributions). We start with a two generations analysis, where a natural geometric interpretation can be applied. It allows us to straightforwardly reproduce known results [7]. We then consider the three generations case, where a dramatic improvement in the measurements related to the top sector is expected at the LHC. Thus, it is rather interesting to assess, before the data is analyzed, what is the potential impact of the projected sensitivity on beyond the SM searches. Our formalism makes manifest the SM approximate  $U(2)$  symmetry, due to the lightness of the first two generation masses, for the up and down quark sectors. In this limit, the SM actually poses a residual  $U(1)_Q$  symmetry, which is automatically incorporated by our formalism. Under this symmetry, the massless first two generations break into an “active” one, which interacts with the heavy state, and a non-interactive “sterile” state. This description is useful, not only conceptually, but also when considering top and jet physics at the LHC, which in practice cannot distinguish between light quark jets. The combination of data from the down and the up sectors is used to robustly constrain models including arbitrary mechanisms of alignment.

The analysis is based on the SM flavor group for quarks:

$$G_{SM} = U(3)_Q \times U(3)_U \times U(3)_D, \quad (1)$$

where  $Q$ ,  $U$  and  $D$  stand for quarks doublets, up-type singlets and down-type singlets, respectively. As mentioned,  $G_{SM}$  is broken within the SM only by the Yukawa interactions. Therefore, we can treat the Yukawa matrices  $Y_u$  and  $Y_d$  as spurions, which transform as  $(\mathbf{3}, \mathbf{\bar{3}}, 1)$  and  $(\mathbf{3}, 1, \mathbf{\bar{3}})$ , respectively, under the flavor group. In order to attain a covariant geometric picture, we need to construct objects out of the Yukawa matrices which transform in the same way. These are simply  $Y_u Y_u^\dagger$  and  $Y_d Y_d^\dagger$ , which are in the  $(\mathbf{8} + 1, 1, 1)$  representation. Since the trace of these matrices does not affect flavor changing processes, it is useful to remove it, and work with  $(Y_u Y_u^\dagger)_{\psi\psi}$  and  $(Y_d Y_d^\dagger)_{\psi\psi}$ , both of which are adjoints of  $U(3)_Q$ . For simplicity of notation, we denote these objects as

$$\mathcal{A}_u \equiv (Y_u Y_u^\dagger)_{\psi\psi}, \quad \mathcal{A}_d \equiv (Y_d Y_d^\dagger)_{\psi\psi}. \quad (2)$$

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<sup>1</sup>Recently, a hint for such a deviation has been observed at the Tevatron in the like-sign dimuon charge asymmetry [5].

As shown below, we can use these SM spurions of flavor violation to construct a covariant basis. This basis turns out to physically describe the flavor violation of the SM, as well as of NP.

This paper is organized as follows: The two generations case, for which a geometric formalism is devised, is discussed in Sec. 2. The covariant description for third generation flavor violation is given in Sec. 3. In Sec. 4 we use our formalism to constrain NP models in an assumption-free manner, based on third generation  $\Delta F = 1$  decays. Sec. 5 similarly deals with  $\Delta F = 2$  processes involving the third generation quarks. For the latter two sections, current experimental data is used for the down sector constraints, while the up sector bounds are mostly based on LHC prospects. Secs. 6 and 7 present concrete examples for the application of the analysis to supersymmetry and warped extra dimension, respectively. Finally, we conclude in Sec. 8.

## 2 Two Generations

We start with the simpler two generations case, which is actually very useful in constraining new physics, as a result of the richer experimental data. Any hermitian traceless  $2 \times 2$  matrix can be expressed as a linear combination of the Pauli matrices  $\sigma_i$ . This combination can be naturally interpreted as a vector in three dimensional real space, which applies to  $\mathcal{A}_d$  and  $\mathcal{A}_u$ . We can then define a length of such a vector, a scalar product, a cross product and an angle between two vectors, all of which are basis-independent<sup>2</sup>:

$$\begin{aligned} |\vec{A}| &\equiv \sqrt{\frac{1}{2}\text{tr}(A^2)}, \quad \vec{A} \cdot \vec{B} \equiv \frac{1}{2}\text{tr}(AB), \quad \vec{A} \times \vec{B} \equiv -\frac{i}{2}[A, B], \\ \cos(\theta_{AB}) &\equiv \frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|} = \frac{\text{tr}(AB)}{\sqrt{\text{tr}(A^2)\text{tr}(B^2)}}. \end{aligned} \quad (3)$$

These definitions allow for an intuitive understanding of the flavor and CP violation induced by a new physics source. Consider a dimension six  $SU(2)_L$ -invariant operator, involving only quark doublets,

$$\frac{z_1}{\Lambda_{\text{NP}}^2} O_1 = \frac{1}{\Lambda_{\text{NP}}^2} (\bar{Q}_i (X_Q)_{ij} \gamma_\mu Q_j) (\bar{Q}_i (X_Q)_{ij} \gamma^\mu Q_j), \quad (4)$$

where  $\Lambda_{\text{NP}}$  is some high energy scale and  $z_1$  is the Wilson coefficient.  $X_Q$  is a traceless hermitian matrix, transforming as an adjoint of  $SU(3)_Q$  (or  $SU(2)_Q$  for two generations), so it “lives” in the same space as  $\mathcal{A}_d$  and  $\mathcal{A}_u$ .<sup>3</sup> In the down sector for example, the operator above is relevant for flavor violation through  $K^0 - \bar{K}^0$  mixing. To analyze its contribution, we define a covariant basis for each sector, with the following unit vectors

$$\hat{\mathcal{A}}_{u,d} \equiv \frac{\mathcal{A}_{u,d}}{|\mathcal{A}_{u,d}|}, \quad \hat{J} \equiv \frac{\mathcal{A}_d \times \mathcal{A}_u}{|\mathcal{A}_d \times \mathcal{A}_u|}, \quad \hat{J}_{u,d} \equiv \hat{\mathcal{A}}_{u,d} \times \hat{J}. \quad (5)$$

Then the contribution of the operator in Eq. (4) to  $\Delta c, s = 2$  processes is given by the misalignment between  $X_Q$  and  $\mathcal{A}_{u,d}$ , which is equal to

$$\left| z_1^{D,K} \right| = \left| X_Q \times \hat{\mathcal{A}}_{u,d} \right|^2. \quad (6)$$

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<sup>2</sup>The factor of  $-i/2$  in the cross product is required in order to have the standard geometrical interpretation  $|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}|\sin\theta_{AB}$ , with  $\theta_{AB}$  defined through the scalar product as in Eq. (3).

<sup>3</sup>This operator can always be written as a product of two identical adjoints, as explained in Appendix A.

This result is manifestly invariant under a change of basis. The meaning of Eq. (6) can be understood as follows: We can choose an explicit basis, for example the down mass basis, where  $\mathcal{A}_d$  is proportional to  $\sigma_3$ .  $\Delta s = 2$  transitions are induced by the off-diagonal element of  $X_Q$ , so that  $|z_1^K| = |(X_Q)_{12}|^2$ . Furthermore,  $|(X_Q)_{12}|$  is simply the combined size of the  $\sigma_1$  and  $\sigma_2$  components of  $X_Q$ . Its size is given by the length of  $X_Q$  times the sine of the angle between  $X_Q$  and  $\mathcal{A}_d$  (see Fig 1). This is exactly what Eq. (6) describes.

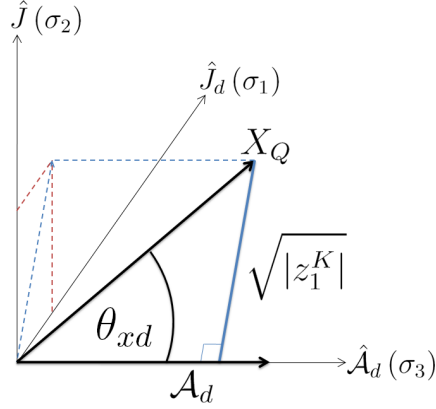


Figure 1: The contribution of  $X_Q$  to  $K^0 - \bar{K}^0$  mixing,  $\Delta m_K$ , given by the solid blue line. In the down mass basis,  $\hat{\mathcal{A}}_d$  corresponds to  $\sigma_3$ ,  $\hat{J}$  is  $\sigma_2$  and  $\hat{J}_d$  is  $\sigma_1$ .

Next we discuss CPV, which is given by

$$\text{Im} \left( z_1^{K,D} \right) = 2 \left( X_Q \cdot \hat{J} \right) \left( X_Q \cdot \hat{J}_{u,d} \right). \quad (7)$$

The above expression is easy to understand in the down basis, for instance. In addition to diagonalizing  $\mathcal{A}_d$ , we can also choose  $\mathcal{A}_u$  to reside in the  $\sigma_1 - \sigma_3$  plane (Fig. 2) without loss of generality, since there is no CPV in the SM for two generations. As a result, all of the potential CPV originates from  $X_Q$  in this basis.  $z_1^K$  is the square of the off-diagonal element in  $X_Q$ ,  $(X_Q)_{12}$ , thus  $\text{Im}(z_1^K)$  is simply twice the real part ( $\sigma_1$  component) times the imaginary part ( $\sigma_2$  component). In this basis we have  $\hat{J} \propto \sigma_1$  and  $\hat{J}_d \propto \sigma_2$ , this proves the validity of Eq. (7).

The weakest unavoidable bound coming from measurements in the  $K$  and  $D$  systems was derived in [7] using a specific parameterization of  $X_Q$ . In the covariant bases defined in Eq. (5),  $X_Q$  can be written as

$$X_Q = X^{u,d} \hat{\mathcal{A}}_{u,d} + X^J \hat{J} + X^{J_{u,d}} \hat{J}_{u,d}, \quad (8)$$

and the two bases are related through

$$X^u = \cos 2\theta_C X^d - \sin 2\theta_C X^{J_d}, \quad X^{J_u} = -\sin 2\theta_C X^d - \cos 2\theta_C X^{J_d}, \quad (9)$$

while  $X^J$  remains invariant. Plugging Eqs. (8) and (9) into Eqs. (6) and (7), we obtain explicit results. It is then easy to see that in the parameterization employed in [7],  $\Lambda_{12} \sin \gamma$  is equal to  $X^J$ ,  $\Lambda_{12} \sin \alpha \cos \gamma$  is equal to  $X^{J_d}$  etc., therefore their results coincide with ours.

An interesting conclusion can be inferred from the analysis above: In addition to the known necessary condition for CPV in two generation [7]

$$X^J \propto \text{tr} (X_Q [\mathcal{A}_d, \mathcal{A}_u]) \neq 0, \quad (10)$$

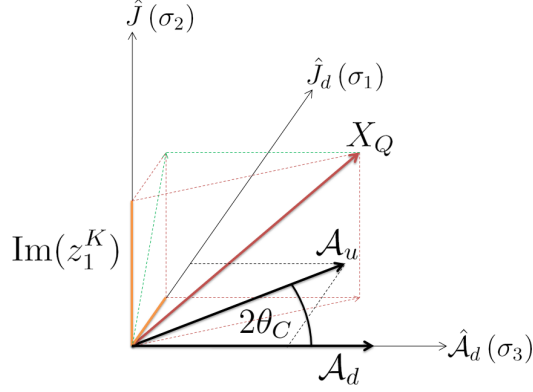


Figure 2: CP violation in the Kaon system induced by  $X_Q$ .  $\text{Im}(z_1^K)$  is twice the product of the two solid orange lines, which are the projections of  $X_Q$  on the  $\hat{J}$  and  $\hat{J}_d$  axes. Note that the angle between  $\mathcal{A}_d$  and  $\mathcal{A}_u$  is twice the Cabibbo angle,  $\theta_C$ .

we identify a second necessary condition, exclusive for  $\Delta F = 2$  processes:

$$X^{J_{u,d}} \propto \text{tr}(X_Q [\mathcal{A}_{u,d}, [\mathcal{A}_d, \mathcal{A}_u]]) \neq 0, \quad (11)$$

The strength of these conditions is that they involve only the basic physical ingredients  $\mathcal{A}_{u,d}$  and  $X_Q$ , and they can be clearly identified from the geometric interpretation. Note, however, that this new condition in Eq. (11) is only applicable to either the down or the up sector, while the known condition in Eq. (10) is universal.

### 3 Three Generations

#### 3.1 Approximate $U(2)$ Limit of Massless Light Quarks

For three generations, a simple 3D geometric interpretation does not naturally emerge anymore, as the relevant space is characterized by the eight Gell-Mann matrices<sup>4</sup>. A useful approximation appropriate for third generation flavor violation is to neglect the masses of the first two generation quarks, where the breaking of the flavor symmetry is characterized by  $[U(3)/U(2)]^2$  [1]. This description is especially suitable for the LHC, where it would be difficult to distinguish between light quark jets of different flavor. In this limit, the 1-2 rotation and the phase of the CKM matrix become unphysical, and we can, for instance, further apply a  $U(2)$  rotation to the first two generations to “undo” the 1-3 rotation. Therefore, the CKM matrix is effectively reduced to a real matrix with a single rotation angle between an active light flavor (say, the 2nd one) and the 3rd generation,

$$\theta \cong \sqrt{\theta_{13}^2 + \theta_{23}^2}, \quad (12)$$

where  $\theta_{13}$  and  $\theta_{23}$  are the corresponding CKM mixing angles. The other generation (the first one) decouples, and is protected by a residual  $U(1)_Q$  symmetry. This can be easily seen when writing

<sup>4</sup>We denote the Gell-Mann matrices by  $\Lambda_i$ , where  $\text{tr}(\Lambda_i \Lambda_j) = 2\delta_{ij}$ . Choosing this convention allows us to keep the definitions of Eq. (3).

$\mathcal{A}_d$  and  $\mathcal{A}_u$  in, say, the down mass basis

$$\mathcal{A}_d = \frac{y_b^2}{3} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}, \quad \mathcal{A}_u = y_t^2 \begin{pmatrix} \spadesuit & 0 & 0 \\ 0 & \spadesuit & \spadesuit \\ 0 & \spadesuit & \spadesuit \end{pmatrix}, \quad (13)$$

where  $\spadesuit$  stands for a non-zero *real* entry. The resulting flavor symmetry breaking scheme is depicted in Fig. 3.

$$\begin{array}{c} U(1)_Q \times U(1)_B \\ \uparrow V_{\text{CKM}} \\ U(2)_Q \times U(1)_{Q_3} \\ \uparrow \mathcal{A}_{u,d} \ (V_{\text{CKM}} \rightarrow \mathbb{1}_3) \\ U(3)_Q \end{array}$$

Figure 3: The flavor symmetry breaking pattern for the left-handed sector: The  $U(3)_Q$  group is broken by each of  $\mathcal{A}_u$  and  $\mathcal{A}_d$  to an approximate  $U(2)_Q \times U(1)_{Q_3}$ ; including the fact that these two objects are not aligned (that is, the CKM matrix is not trivial), the symmetry is broken into baryon number and an approximate  $U(1)_Q$  for the light quarks combination that effectively decouples.

An interesting consequence of this approximation is that a complete basis cannot be defined covariantly, since  $\mathcal{A}_{u,d}$  in Eq. (13) clearly span only a part of the eight dimensional space. More concretely, we can identify four directions in this space:  $\hat{J}$  and  $\hat{J}_{u,d}$  from Eq. (5) and either one of the two orthogonal pairs

$$\hat{\mathcal{A}}_{u,d} \quad \text{and} \quad \hat{C}_{u,d} \equiv 2\hat{J} \times \hat{J}_{u,d} - \sqrt{3}\hat{\mathcal{A}}_{u,d}, \quad (14)$$

or

$$\hat{\mathcal{A}}'_{u,d} \equiv \hat{J} \times \hat{J}_{u,d} \quad \text{and} \quad \hat{J}_Q \equiv \sqrt{3}\hat{J} \times \hat{J}_{u,d} - 2\hat{\mathcal{A}}_{u,d}. \quad (15)$$

Note that  $\hat{J}_Q$  corresponds to the conserved  $U(1)_Q$  generator, so it commutes with both  $\mathcal{A}_d$  and  $\mathcal{A}_u$ , and takes the same form in both bases<sup>5</sup>. There are four additional directions, collectively denoted as  $\hat{\vec{D}}$ , which transform as a doublet under the CKM (2-3) rotation, and do not mix with the other generators. The fact that these cannot be written as combinations of  $\mathcal{A}_{u,d}$  stems from the approximation introduced above of neglecting light quark masses. Without this assumption, it is possible to span the entire space using the Yukawa matrices [8, 9, 10]. Despite the fact that this can be done in several ways, in the next subsection we focus on a realization for which the basis elements have a clear physical meaning.

<sup>5</sup>The meaning of these basis elements can be understood from the following: In the down mass basis we have  $\hat{\mathcal{A}}_d = -\Lambda_8$ ,  $\hat{J} = \Lambda_7$ ,  $\hat{J}_d = \Lambda_6$  and  $\hat{C}_d = \Lambda_3$ . The alternative diagonal generators from Eq. (15) are  $\hat{\mathcal{A}}'_d = (\Lambda_3 - \sqrt{3}\Lambda_8)/2 = \text{diag}(0, -1, 1)$  and  $\hat{J}_Q = (\sqrt{3}\Lambda_3 + \Lambda_8)/2 = \text{diag}(2, -1, -1)/\sqrt{3}$ . It is then easy to see that  $\hat{J}_Q$  commutes with the effective CKM matrix, which is just a 2-3 rotation, and that it corresponds to the  $U(1)_Q$  generator,  $\text{diag}(1, 0, 0)$ , after trace subtraction and proper normalization.

It is interesting to notice that a given traceless adjoint object  $X$  in three generations flavor space has an inherent  $SU(2)$  symmetry (that is, two identical eigenvalues) if and only if it satisfies

$$[\text{tr}(X^2)]^{3/2} = \sqrt{6} \text{tr}(X^3) . \quad (16)$$

In this case it must be a unitary rotation of either  $\Lambda_8$  or its permutations  $(\Lambda_8 \pm \sqrt{3}\Lambda_3)/2$ , which form an equilateral triangle in the  $\Lambda_3 - \Lambda_8$  plane (see Fig. (4)).

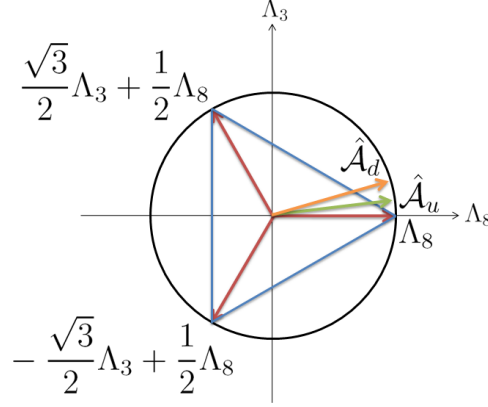


Figure 4: The three unit-length diagonal traceless matrices with an inherent  $SU(2)$  symmetry.  $\hat{\mathcal{A}}_d$  and  $\hat{\mathcal{A}}_u$  were schematically added (their angle to the  $\Lambda_8$  axis is actually much smaller than what appears in the plot).

As before, we wish to characterize the flavor violation induced by  $X_Q$  in a basis independent form. The simplest observable we can construct is the overall flavor violation of the third generation quark, that is, its decay to any quark of the first two generations. This can be written as

$$\frac{2}{\sqrt{3}} \left| X_Q \times \hat{\mathcal{A}}_{u,d} \right| , \quad (17)$$

which extracts  $\sqrt{|(X_Q)_{13}|^2 + |(X_Q)_{23}|^2}$  in each basis.

### 3.2 No $U(2)$ Limit – Complete Covariant Basis

It is sufficient to restore the masses of the second generation quarks in order to describe the full flavor space. A simplifying step to accomplish this is to define the following object: We take the  $n$ -th power of  $(Y_d Y_d^\dagger)$ , remove the trace, normalize and take the limit  $n \rightarrow \infty$ . This is denoted by  $\hat{\mathcal{A}}_d^n$ :

$$\hat{\mathcal{A}}_d^n \equiv \lim_{n \rightarrow \infty} \left\{ \frac{(Y_d Y_d^\dagger)^n - \mathbf{1}_3 \text{tr}[(Y_d Y_d^\dagger)^n]/3}{|(Y_d Y_d^\dagger)^n - \mathbf{1}_3 \text{tr}[(Y_d Y_d^\dagger)^n]/3|} \right\} , \quad (18)$$

and we similarly define  $\hat{\mathcal{A}}_u^n$ . Once we take the limit  $n \rightarrow \infty$  the small eigenvalues of  $\hat{\mathcal{A}}_{u,d}$  go to zero and the approximation assumed before is formally reproduced. As before, we compose the following basis elements:

$$\hat{J}^n \equiv \frac{\hat{\mathcal{A}}_d^n \times \hat{\mathcal{A}}_u^n}{|\hat{\mathcal{A}}_d^n \times \hat{\mathcal{A}}_u^n|} , \quad \hat{J}_d^n \equiv \frac{\hat{\mathcal{A}}_d^n \times \hat{J}^n}{|\hat{\mathcal{A}}_d^n \times \hat{J}^n|} , \quad \hat{C}_d^n \equiv 2\hat{J}^n \times \hat{J}_d^n - \sqrt{3}\hat{\mathcal{A}}_d^n , \quad (19)$$



which are again identical to the previous case. The important observation for this case is that the  $U(1)_Q$  symmetry is now broken. Consequently, the  $U(1)_Q$  generator,  $J_Q$ , does not commute with  $\mathcal{A}_d$  and  $\mathcal{A}_u$  anymore (nor does  $\hat{C}_d^n$ , which is different from  $J_Q$  only by normalization and a shift by  $\mathcal{A}_d$ , see Eqs. (14) and (15)). It is thus expected that the commutation relation  $[\mathcal{A}_d, \hat{C}_d^n]$  (where  $\mathcal{A}_d$  now contains also the strange quark mass) would point to a new direction, which could not be obtained in the approximation used before. Further commutations with the existing basis elements should complete the description of the flavor space.

We thus define

$$\hat{D}_2 \equiv \frac{\hat{\mathcal{A}}_d \times \hat{C}_d^m}{\left| \hat{\mathcal{A}}_d \times \hat{C}_d^m \right|}. \quad (20)$$

In order to understand the physical interpretation, note that  $\hat{D}_2$  does not commute with  $\mathcal{A}_d$ , so it must induce flavor violation, yet it does commute with  $\hat{\mathcal{A}}_d^n$ . The latter can be identified as a generator of a  $U(1)$  symmetry for the bottom quark (it is proportional to  $\text{diag}(0,0,1)$  in its diagonal form, without removing the trace), so this fact means that  $\hat{D}_2$  preserves this symmetry. Therefore it must represent a transition between the first two generations of the down sector.

We further define

$$\hat{D}_1 \equiv \frac{\hat{\mathcal{A}}_d \times \hat{D}_2}{\left| \hat{\mathcal{A}}_d \times \hat{D}_2 \right|}, \quad \hat{D}_4 \equiv \frac{\hat{J}_d^n \times \hat{D}_2}{\left| \hat{J}_d^n \times \hat{D}_2 \right|}, \quad \hat{D}_5 \equiv \frac{\hat{J}^n \times \hat{D}_2}{\left| \hat{J}^n \times \hat{D}_2 \right|}, \quad (21)$$

which complete the basis. All of these do not commute with  $\mathcal{A}_d$ , thus producing down flavor violation.  $\hat{D}_1$  commutes with  $\hat{\mathcal{A}}_d^n$ , so it is of the same status as  $\hat{D}_2$ . The last two elements,  $\hat{D}_{4,5}$ , are responsible for third generation decays, similarly to  $\hat{J}^n$  and  $\hat{J}_d^n$ . More concretely, the latter two involve transitions between the third generation and what was previously referred to as the “active” generation (a linear combination of the first two), while  $\hat{D}_{4,5}$  mediate transitions to the orthogonal combination. It is of course possible to define linear combinations of these four basis elements, such that the decays to the strange and the down mass eigenstates are separated, but we do not proceed with this derivation. It is also important to note that this basis is not completely orthogonal. An explicit decomposition of all the covariant objects in a specific basis can be found in Appendix B.

An instructive exercise is to decompose  $\mathcal{A}_u$  in this covariant “down” basis, since  $\mathcal{A}_u$  is a flavor violating source within the SM. Focusing only on the dependence on the small parameters  $\lambda_C$  and  $y_c^2/y_t^2$  (and omitting for simplicity  $\mathcal{O}(1)$  factors such as the Wolfenstein parameter  $A$ ), we have

$$\mathcal{A}_u \cdot \left\{ \hat{D}_1, \hat{D}_2, \hat{C}_d^m, \hat{D}_4, \hat{D}_5, \hat{J}_d^n, \hat{J}^n, \hat{\mathcal{A}}_d^n \right\} \sim \left\{ \lambda_C y_c^2 + \lambda_C^5 y_t^2, \lambda_C y_c^2, (y_c^2 + \lambda_C^4 y_t^2)/2, \lambda_C^3 y_c^2, \lambda_C^3 y_c^2, \lambda_C^2 y_t^2, 0, y_t^2/\sqrt{3} \right\}. \quad (22)$$

This shows the different types of flavor violation in the down sector within the SM. It should be mentioned that the  $\hat{D}_2$  and  $\hat{D}_5$  projections of  $\mathcal{A}_u$  vanish when the CKM phase is taken to zero, and also when either of the CKM mixing angles is zero or  $\pi/2$ . Therefore these basis elements can be interpreted as CP violating, together with  $\hat{J}^n$ . As an example, notice that a  $2 \rightarrow 1$  transition in the down sector, represented by the projection to  $\hat{D}_1$ , can either occur via mixing with the third generation at the order  $\lambda_C^5 y_t^2$  or among the first two generations only at the order  $\lambda_C y_c^2$ . Yet CPV in this transition can only be generated through the latter type of contribution at  $\lambda_C y_c^2$ , as can be seen from the  $\hat{D}_2$  projection (recall again that these are not the  $d$  and  $s$  mass eigenstates, but instead



the “active” and “inactive” generations, after a  $U(2)$  rotation has been applied). Analogously, a  $3 \rightarrow 1$  transition occurs at  $\lambda_C^3 y_c^2$  whether it is CP conserving or CP violating, as inferred from the  $\hat{D}_{4,5}$  projections.

In the rest of the paper we use the description based on the approximate  $U(2)$  symmetry, rather than the full basis, whenever possible.

## 4 Third Generation $\Delta F = 1$ Transitions

We now use measurements from the down and the up sectors to derive a model independent bound on the corresponding NP scale, based on the overall flavor violating decay of the third generation quarks. We focus on the following operator

$$O_{LL}^h = i [\bar{Q}_i \gamma^\mu (X_Q^{\Delta F=1})_{ij} Q_j] \left[ H^\dagger \overleftrightarrow{D}_\mu H \right] + \text{h.c.}, \quad (23)$$

which contributes at tree level to both top and bottom decays [11]<sup>6</sup>. Note that as in the two generations case, we only deal with the left-handed sector, where down and up contributions are related. We omit an additional operator for quark doublets,  $O_{LL}^u = i [\bar{Q}_3 \tilde{H}] \left[ (\not{D} \tilde{H})^\dagger Q_2 \right] - i [\bar{Q}_3 (\not{D} \tilde{H})] \left[ \tilde{H}^\dagger Q_2 \right]$ , which induces bottom decays only at one loop, but in principle it should be included in a more detailed analysis.

The experimental constraints we use are [13, 14, 15]

$$\begin{aligned} \text{Br}(B \rightarrow X_s \ell^+ \ell^-)_{1 \text{ GeV}^2 < q^2 < 6 \text{ GeV}^2} &= (1.61 \pm 0.51) \times 10^{-6}, \\ \text{Br}(t \rightarrow (c, u)Z) &< 5.5 \times 10^{-5}, \end{aligned} \quad (24)$$

where the latter corresponds to the prospect of the LHC bound in the absence of signal for  $100 \text{ fb}^{-1}$ . We adopt the weakest limits on the coefficient of the operator in Eq. (23),  $C_{LL}^h$ , derived in [11]:

$$\begin{aligned} \text{Br}(B \rightarrow X_s \ell^+ \ell^-) &\longrightarrow |C_{LL}^h|_b < 0.018 \left( \frac{\Lambda_{\text{NP}}}{1 \text{ TeV}} \right)^2, \\ \text{Br}(t \rightarrow (c, u)Z) &\longrightarrow |C_{LL}^h|_t < 0.18 \left( \frac{\Lambda_{\text{NP}}}{1 \text{ TeV}} \right)^2, \end{aligned} \quad (25)$$

and define  $r_{tb} \equiv |C_{LL}^h|_t / |C_{LL}^h|_b$ .

The NP contribution can be decomposed in the covariant bases as

$$X_Q^{\Delta F=1} = X'^{u,d} \hat{\mathcal{A}}'_{u,d} + X^J \hat{J} + X^{J_{u,d}} \hat{J}_{u,d} + X^{J_Q} \hat{J}_Q + X^{\vec{D}} \hat{\vec{D}}. \quad (26)$$

The length of  $X_Q^{\Delta F=1}$  is denoted, based on the definition in Eq. (3), by

$$L \equiv |X_Q^{\Delta F=1}|. \quad (27)$$

The weakest bound is obtained, for a fixed  $L$ , by finding a direction of  $X_Q$  that minimizes the contributions to  $|C_{LL}^h|_t$  and  $|C_{LL}^h|_b$ , thus constituting the “best” alignment. However, since  $\hat{J}_Q$  commutes with  $\mathcal{A}_{u,d}$ , as discussed above, it does not contribute to third generation decay (Eq. (17))

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<sup>6</sup>It is important to note that a given NP model might generate different higher-dimensional operators via different types of processes (the general form of the relevant low energy effective theory is discussed for example in [12]). Therefore  $X_Q$  is in general different for each operator, so we denote it specifically as  $X_Q^{\Delta F=1}$  for the current case.

in neither sectors. On the other hand, any component of  $X_Q^{\Delta F=1}$  may also generate flavor violation among the first two generations (when their masses are switched back on), which is more strongly constrained. Specifically, the bound that stems from the case of  $X_Q^{\Delta F=1} \propto \hat{J}_Q$ , derived in Appendix C, is

$$L < 0.59 \left( \frac{\Lambda_{\text{NP}}}{1 \text{ TeV}} \right)^2 ; \quad \Lambda_{\text{NP}} > 1.7 \text{ TeV}, \quad (28)$$

where the latter is for  $L = 1$ . This is stronger than the limit given below for other forms of  $X_Q^{\Delta F=1}$ , hence this does not constitute the optimal alignment. To conclude this issue, all directions that contribute to first two generations flavor and CPV at  $\mathcal{O}(\lambda_C)$ , that is  $\hat{J}_Q$ ,  $\hat{\mathcal{D}}$  and  $\hat{\mathcal{A}}_{u,d}$ , are not favorable in terms of alignment, as discussed in Appendix C.

The induced third generation flavor violation, after removing these contributions, is then given by

$$\frac{4}{3} \left| X_Q^{\Delta F=1} \times \hat{\mathcal{A}}_{u,d} \right|^2 = (X^J)^2 + (X^{J_{u,d}})^2, \quad (29)$$

and in order to see this in a common basis, we express  $X^{J_u}$  as

$$X^{J_u} = \cos 2\theta X^{J_d} + \sin 2\theta X'^d, \quad (30)$$

with  $\theta$  as defined in Eq. (12). From this it is clear that  $X^J$  contributes the same to both the top and the bottom decay rates, so it should be set to zero for optimal alignment. Thus the best alignment is obtained by varying  $\alpha$ , defined by

$$\tan \alpha \equiv \frac{X^{J_d}}{X^d}. \quad (31)$$

Here we use  $X^d$ , which is the coefficient of  $\hat{\mathcal{A}}_d$ , instead of  $X'^d$ , since the former does not produce flavor violation among the first two generations to leading order (up to  $\mathcal{O}(\lambda_C^5)$ ).

We now consider two possibilities: (i) complete alignment with the down sector; (ii) the best alignment satisfying the bounds of Eq. (25), which gives the weakest unavoidable limit. Note that we can also consider up alignment, but it would give a stronger bound than down alignment, as a result of the stronger experimental constraints. The bounds for these cases are [4]

$$\begin{aligned} \text{(i)} \quad & \alpha = 0, \quad L < 2.5 \left( \frac{\Lambda_{\text{NP}}}{1 \text{ TeV}} \right)^2 ; \quad \Lambda_{\text{NP}} > 0.63 (7.9) \text{ TeV}, \\ \text{(ii)} \quad & \alpha = \frac{\sqrt{3}\theta}{1 + r_{tb}}, \quad L < 2.8 \left( \frac{\Lambda_{\text{NP}}}{1 \text{ TeV}} \right)^2 ; \quad \Lambda_{\text{NP}} > 0.6 (7.6) \text{ TeV}, \end{aligned} \quad (32)$$

as shown in Fig. 5, where in parentheses we give the strong coupling bound, in which the coefficient of the operators in Eqs. (4) and (23) is assumed to be  $16\pi^2$ . Note that these are weaker than the bound in Eq. (28).

It is important to mention that the optimized form of  $X_Q^{\Delta F=1}$  generates also  $c \rightarrow u$  decay at higher order in  $\lambda_C$ , which might yield stronger constraints than the top decay. In Appendix C it is shown that the resulting bound from the former is actually much weaker than the one from the top. Therefore, the LHC is indeed expected to strengthen the model independent constraints.

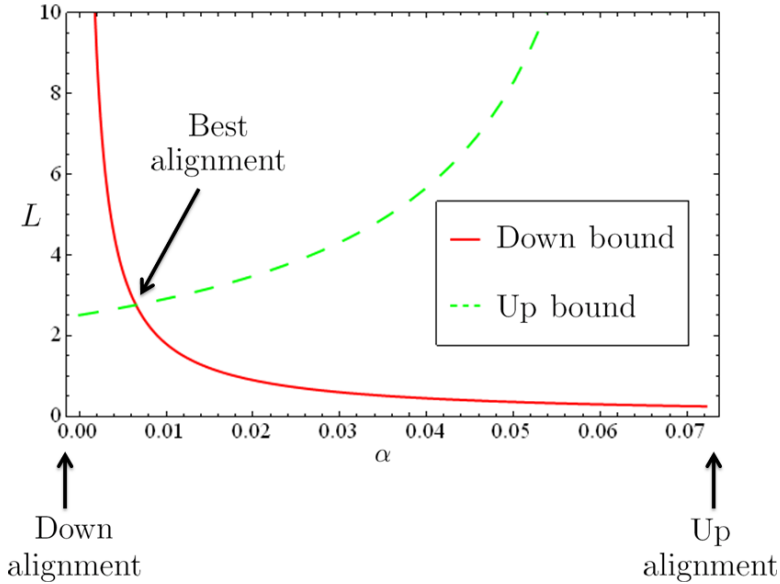


Figure 5: Upper bounds on  $L$  as a function of  $\alpha$ , coming from the measurements of flavor violating decays of the bottom and the top quarks, assuming  $\Lambda_{\text{NP}} = 1$  TeV.

## 5 Third Generation $\Delta F = 2$ Transitions

In the previous section we used  $\Delta F = 1$  decays of third generation quarks to obtain the weakest model independent constraint. Here we do the same using  $\Delta F = 2$  processes. For simplicity, we only consider complete alignment with the down sector

$$X_Q^{\Delta F=2} = L \hat{\mathcal{A}}_d, \quad (33)$$

as the constraints from this sector are much stronger. This generates in the up sector top flavor violation, and also  $D^0 - \bar{D}^0$  mixing at higher order. Yet there is no top meson, as the top quark decays too rapidly to hadronize. Instead, we analyze the process  $pp \rightarrow tt$  (related to mixing by crossing symmetry), which is most appropriate for the LHC. It should be emphasized, however, that in this case the parton distribution functions of the proton strongly break the approximate  $U(2)$  symmetry of the first two generations. The simple covariant basis introduced in Sec. 3.1, which is based on this approximate symmetry, cannot be used as a result. Furthermore, this LHC process is dominated by  $uu \rightarrow tt$ , so we focus only on the operator involving up (and not charm) quarks. We verified numerically that indeed the charm contribution to this process is smaller by an order of magnitude.

The production of same-sign tops was studied in the literature in the context of different models (see *e.g.* [16, 17, 18] and refs. therein). The simplest way observe it at the LHC and distinguish it from  $t\bar{t}$  production, is based on the dilepton mode, in which two same-sign (mostly positive sign) leptons are produced from the top quarks. However, the branching ratio of this mode is only about 5%, and there are several types of SM backgrounds, such as  $W^+W^+qq$ . We therefore choose to adopt a realistic assumption of 1% efficiency for detecting same-sign tops at the LHC [16], including b-tagging efficiency and the necessary cuts to isolate the signal<sup>7</sup>. In any case, our conclusions are only mildly sensitive to this assumption, as explained below.

<sup>7</sup>Examples for possible cuts are requiring some minimal invariant mass for one or two pairings of a lepton and a b-tagged jet and a minimal transverse momentum for the latter jets. The chosen cuts strongly affect the efficiency – in [18], *e.g.* they eliminate the background almost completely, but at the cost of reducing the signal cross section

In order to estimate the prospect for the LHC bound on same-sign tops production, we calculated the  $uu \rightarrow tt$  cross section using MadGraph/MadEvent [19], as a  $t$  (or  $u$ ) channel process mediated by a heavy vector boson, the mass of which is identified with  $\Lambda_{\text{NP}}$ . The resulting cross section for the LHC with center of mass energy of 14, 10 and 7 TeV, and for the Tevatron, is given by

$$\sigma^{tt} = \{60, 30, 13, 0.013\} \left( \frac{1 \text{ TeV}}{\Lambda_{\text{NP}}} \right)^4 \text{ pb} \quad (34)$$

respectively<sup>8</sup>. This was matched onto the operator in Eq. (4). We then used the fact that the cross section times the integrated luminosity must be lower than 3 for a 95% exclusion, in the absence of signal [20]. Adding the assumption of 1% signal efficiency, we find

$$z_1^{tt} < 7.1 \times 10^{-3} \left( \frac{\Lambda_{\text{NP}}}{1 \text{ TeV}} \right)^2, \quad (35)$$

for  $100 \text{ fb}^{-1}$  at a center of mass energy of 14 TeV. The experimental constraint from CPV in the  $D$  system is [21, 22]

$$\text{Im}(z_1^D) < 1.1 \times 10^{-7} \left( \frac{\Lambda_{\text{NP}}}{1 \text{ TeV}} \right)^2, \quad (36)$$

The contribution of  $X_Q^{\Delta F=2}$  to these processes is calculated by applying a CKM rotation to Eq. (33). CPV in the  $D$  system is then given by  $\text{Im} \left[ (X_Q^{\Delta F=2})_{12}^2 \right]$ , and  $\left| (X_Q^{\Delta F=2})_{13} \right|^2$  describes  $uu \rightarrow tt$ . Note that we have

$$\begin{aligned} (X_Q^{\Delta F=2})_{12} &\cong -\sqrt{3} LV_{ub} V_{cb}^*, \\ (X_Q^{\Delta F=2})_{13} &\cong -\sqrt{3} LV_{ub} V_{tb}^*, \end{aligned} \quad (37)$$

with  $V_{ij}$  as the CKM matrix elements. The resulting bounds are

$$L < 12 \left( \frac{\Lambda_{\text{NP}}}{1 \text{ TeV}} \right); \quad \Lambda_{\text{NP}} > 0.08 (1) \text{ TeV}, \quad (38)$$

for  $uu \rightarrow tt$  and

$$L < 1.8 \left( \frac{\Lambda_{\text{NP}}}{1 \text{ TeV}} \right); \quad \Lambda_{\text{NP}} > 0.57 (7.2) \text{ TeV}, \quad (39)$$

for  $D$  mixing. It should be mentioned that the bound in Eq. (38) depends on the quartic root of the cross section that was evaluated above, thus it is only mildly sensitive to that calculation and to the efficiency assumption. Interestingly, the bound that stems from the Tevatron with  $5 \text{ fb}^{-1}$  (assuming that same-sign top pairs were searched for and not detected) is weaker than Eq. (38) by a factor of  $\sim 17$ .

The limits in Eqs. (38) and (39) can be further weakened by optimizing the alignment between the down and the up sectors, as in the previous section. Since this would only yield a marginal improvement of about 10%, we do not analyze this case in detail.

To conclude, we learn that for  $\Delta F = 2$  processes, the existing bound is stronger than the one which will be obtained at the LHC for top quarks, as opposed to  $\Delta F = 1$  case considered above.

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to less than 0.2% of its original value. A detailed analysis of this issue, which can be found in the literature, is out of the scope of the paper.

<sup>8</sup>The simulation was actually performed with a high mass for the new vector boson, to avoid producing it on-shell. The result was then scaled down to 1 TeV as  $\Lambda_{\text{NP}}^4$  (we also verified within the simulation that this scaling is correct).

## 6 Supersymmetry

The analysis presented above uses a model-independent language via effective field theory. Here we apply our results to two SM extensions – supersymmetry (SUSY) and the Randall-Sundrum (RS) model of a warped extra dimension (in the next section).

We focus now on both  $\Delta F = 1$  and  $\Delta F = 2$  left-handed processes within supersymmetric extensions of the SM. The idea as in the above is to provide robust bounds, which could be applied even to SUSY alignment models [23] (for a possible connection with bounds from EDM see [24]). The analysis of  $\Delta F = 1$  transitions is more involved as follows. The relevant contributions to the left-handed operators is driven by the squark doublets mass matrix, which transforms as an adjoint of the minimal supersymmetric standard model (MSSM) flavor group. However, the contributions to top and bottom decays are induced by different operators in the effective Hamiltonian, hence our treatment above does not apply. Instead we rederive the relevant bounds on the squark mass matrix explicitly.

Given the large number of parameters involved in flavor changing processes, it is often convenient to use the mass insertion (MI) formalism. The mass insertions are defined in the so-called super CKM basis. In this basis all the neutral gaugino couplings  $\tilde{g}, \tilde{\gamma}, \tilde{Z}$  are flavor diagonal, and the charged  $\tilde{W}^\pm$  quark-squark mixing angles are equal to the CKM angles. In general, the squarks mass matrices  $\tilde{m}^{u,d}$  are not diagonal in the Super CKM basis. Flavor violation is induced by the  $\tilde{m}^{u,d}$  off diagonal elements, and can be parameterized in terms of the ratios

$$\left(\delta_{ij}^f\right)_{AB} = \frac{\left(\tilde{m}_{ij}^f\right)_{AB}^2}{\tilde{m}_Q^2}, \quad (40)$$

where  $\left(\tilde{m}_{ij}^f\right)_{AB}^2$  are the off-diagonal elements of the  $f = u, d$  mass squared matrix that mixes flavors  $i, j$  for both left- and right-handed scalars ( $A, B = \text{Left, Right}$ ), and where  $\tilde{m}_Q$  indicates the average squark mass.

### 6.1 Top Decay

In the computations of the branching ratio  $\mathcal{B}(t \rightarrow cZ)$  we follow the analysis of [25] (see also [26]). We first work in the basis where the squarks mass matrix is diagonal. In this basis the diagrams relevant for the  $t \rightarrow cZ$  process are shown in Fig. (6).

The effective vertex relevant for FCNC can be parameterized as

$$-i\bar{u}(p) \left[ P_R \left( F_L^a q^2 \gamma^\mu + F_L^b \not{q} q^\mu + G_L i \sigma^{\mu\nu} q_\nu \right) + P_L \left( F_R^a q^2 \gamma^\mu + F_R^b \not{q} q^\mu + G_R i \sigma^{\mu\nu} q_\nu \right) \right] \epsilon_\mu u(p+q), \quad (41)$$

where  $P_{L,R} = \frac{(1 \mp \gamma_5)}{2}$ . The form factors  $F^a$  can be in general written as

$$F_L^a = \frac{g_s^2}{4\pi^2} \sum_{\alpha, \beta=1}^6 \left[ K_{\tilde{c}_L, \beta}^\dagger F_{1L}^a(\alpha, \beta) K_{\alpha, \tilde{t}_L} - K_{\tilde{c}_L, \beta}^\dagger F_{2L}^a(\alpha, \beta) K_{\alpha, \tilde{t}_R} \right], \quad (42)$$

$$F_R^a = \frac{g_s^2}{4\pi^2} \sum_{\alpha, \beta=1}^6 \left[ K_{\tilde{c}_R, \beta}^\dagger F_{1R}^a(\alpha, \beta) K_{\alpha, \tilde{t}_R} - K_{\tilde{c}_R, \beta}^\dagger F_{2R}^a(\alpha, \beta) K_{\alpha, \tilde{t}_L} \right], \quad (43)$$

where the indices  $\alpha, \beta$  identify the squarks mass eigenstates and  $K_{\alpha\beta}$  is the matrix that diagonalizes the squarks mass matrix. Moreover, it can be shown that analogous expressions are valid for  $F^b$  and  $G$  from Eq. (41).

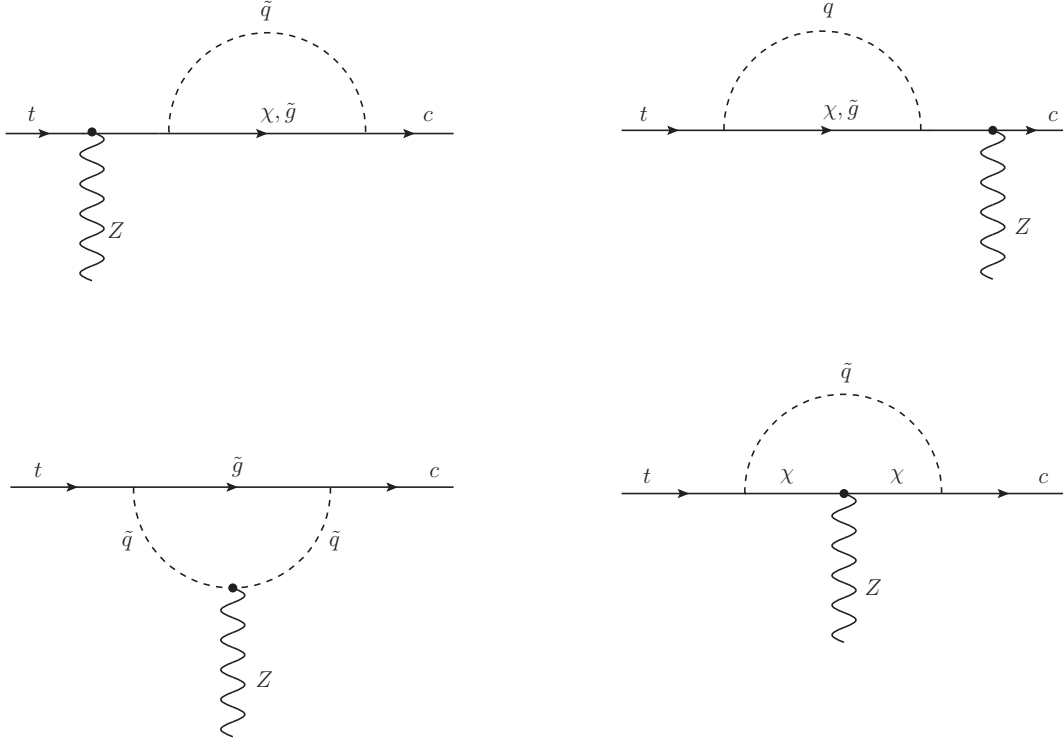


Figure 6: Feynman diagrams for the process  $t \rightarrow cZ$ . The dashed lines represent squarks exchange.

Given that we perform a model independent analysis, we choose the model parameters in order to obtain a robust bound on  $(\delta_{23})_{LL}$ . Thus we set  $(\delta_{23})_{RL} = 0$  in the squarks mass matrix. In this case the only contribution to  $(\delta_{23})_{LL}$  comes from

$$F_{1L}^a(\alpha, \beta) = a_L^{qqZ} \delta_{\alpha\beta} C_F \frac{1}{2q^2} [B_x(m_t^2, m_{\tilde{g}}^2, m_\alpha^2) + B_0(m_t^2, m_\alpha^2, m_{\tilde{g}}^2)] , \quad (44)$$

where  $m_t$ ,  $m_{\tilde{g}}$  and  $m_\alpha$  correspond respectively to the top, gluino and squarks masses. Furthermore we introduce the definitions  $C_F = \frac{N^2-1}{2N} = \frac{4}{3}$  and

$$a_L^{qqZ} = \frac{g_w}{2 \cos(\theta_w)} \left[ -1 + \frac{4}{3} \sin^2(\theta_w) \right] . \quad (45)$$

The complete expression for the form factors, including the expressions for the decay into photons and gluons can be found in [25]. The quantities  $B_x$  and  $B_0$  in (44) are given by

$$\begin{aligned} B_0(p^2, m_1^2, m_2^2) &= \text{pole terms} - \int_0^1 d\alpha \log [p^2 \alpha^2 + (m_1^2 - m_2^2 - p^2) \alpha + m_2^2] , \\ B_x(p^2, m_1^2, m_2^2) &= \text{pole terms} + \int_0^1 d\alpha \alpha \log [p^2 \alpha^2 + (m_1^2 - m_2^2 - p^2) \alpha + m_2^2] , \end{aligned} \quad (46)$$

where the divergent parts in the above integrals cancel in the final result. In the following we work in the approximation of quasi-degeneracy for the squarks, thus moving to the super-CKM basis. Expanding in terms of the mass insertions  $(\delta_{ij}^u)_{LL}$ , we arrive at the following expression for the part of the form factors contributing to flavor violation

$$(F_L)_{ij} \big|_{q^2=m_Z^2} = \frac{g_s^2}{4\pi^2} a_L^{qqZ} C_F \frac{1}{2m_Z^2} f(x_t, x_g) (\delta_{ij}^u)_{LL} , \quad (47)$$

with  $f(x_t, x_g)$  given by

$$f(x_t, x_g) \equiv \frac{1}{2x_t^2 h(x_t, x_g)} \left\{ h(x_t, x_g) [(x_g + x_t - 1) \log(x_g) - 2x_t] \right. \\ \left. + 2\sqrt{h(x_t, x_g)} [(x_g - 2)x_g + (x_t - 1)^2] \tan^{-1} \left( \frac{x_g - x_t - 1}{\sqrt{h(x_t, x_g)}} \right) \right. \\ \left. - 2\sqrt{h(x_t, x_g)} [(x_g - 2)x_g + (x_t - 1)^2] \tan^{-1} \left( \frac{x_g + x_t - 1}{\sqrt{h(x_t, x_g)}} \right) \right\}, \quad (48)$$

and where we use

$$x_g \equiv \frac{m_{\tilde{g}}^2}{\tilde{m}_Q^2}, \quad x_t \equiv \frac{m_t^2}{\tilde{m}_Q^2}, \quad h(x_t, x_g) \equiv 2x_g(x_t + 1) - x_g^2 - (x_t - 1)^2. \quad (49)$$

In terms of the form factors, the expression for the branching ratio, normalized to  $\Gamma(t \rightarrow b + W)$ , is given by

$$\mathcal{B}(t \rightarrow cZ) = \frac{1}{\Gamma(t \rightarrow b + W)} \frac{1}{32\pi} \frac{m_t^2 - m_Z^2}{2m_t^3} (F_L^a)^2|_{q^2=m_Z^2} (2m_t^4 m_Z^2 + 2m_t^2 m_Z^4 - 4m_Z^6). \quad (50)$$

Evaluating Eq. (50) at  $\tilde{m}_Q = 100$  GeV and  $\tilde{m}_Q = m_{\tilde{g}}$ , we get the following bound for  $(\delta_{23}^u)_{LL}$

$$(\delta_{23}^u)_{LL} < 0.84. \quad (51)$$

## 6.2 Bottom Decay

We now move to discuss the  $b \rightarrow s\ell^+\ell^-$  transition. The branching ratio is given in terms of the Wilson coefficients by [11]

$$\mathcal{B}(B \rightarrow X_s \ell^+ \ell^-)_{1 < q^2 < 6 \text{ GeV}^2} = 10^{-6} \left\{ 1.55 + 35100 [|\Delta C_9(m_W)|^2 + |\Delta C_{10}(m_W)|^2] \right. \\ \left. + \text{Re}[(180 + 5i) \Delta C_9(m_W)] - 360 \text{Re}[\Delta C_{10}(m_W)] \right\}. \quad (52)$$

The contributions to flavor violation coming from the MSSM can be derived in the MI approximation. In order to obtain a robust bound on  $(\delta_{23}^d)_{LL}$ , we neglect the chargino contributions, which depend on additional parameters, such as  $\mu$ ,  $\tan \beta$  etc.. Under this assumption, the explicit values for the MSSM contributions to  $C_9$  with  $\tilde{m}_Q = 100$  GeV and  $\tilde{m}_Q = m_{\tilde{g}}$  are

$$\Delta C_9(m_W) = -1.75 (\delta_{23}^d)_{LL}, \quad (53)$$

and  $C_{10}$  vanishes, see [27]<sup>9</sup> (and similar Refs. [28]).

Combining Eqs. (52) and (53) with the experimental bound in Eq. (24), we obtain the following bound

$$(\delta_{23}^d)_{LL} < 0.003. \quad (54)$$

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<sup>9</sup>Note that there is a factor of 2 between the definitions of the operator  $O_9$  in [27] and [11]



### 6.3 Best alignment

As a result of the large difference between the top and bottom bounds, Eqs. (51) and (54), the best alignment scenario is practically equivalent to alignment with the down sector. In this case,  $(\delta_{23}^d)_{LL} = 0$  and  $(\delta_{23}^u)_{LL}$  is simply proportional to the squarks mass squared difference multiplied by  $V_{cb}V_{tb}^*$ . Taking for concreteness  $\tilde{m}_Q = (2m_{\tilde{Q}_2} + m_{\tilde{Q}_3})/3$  (appropriate for models with only weak degeneracy [29]), the bound is then

$$\frac{|m_{\tilde{Q}_2}^2 - m_{\tilde{Q}_3}^2|}{(2m_{\tilde{Q}_2} + m_{\tilde{Q}_3})^2} < 20 \left( \frac{\tilde{m}_Q}{100 \text{ GeV}} \right)^2. \quad (55)$$

We find therefore that no significant constraint on the level of degeneracy can be obtained.

### 6.4 $\Delta F = 2$ Processes

We next move to describe  $\Delta F = 2$  processes, for which the results of Sec. 5 are easily applied. Considering again the leading order in the expansion  $(\delta_{13})_{LL}$ , we arrive at the following expression for the length of  $X_Q$

$$L = \frac{\alpha_s}{18} \sqrt{\frac{g(x)}{2}} (\delta_{13})_{LL}, \quad (56)$$

where  $x = m_{\tilde{g}}^2/\tilde{m}_Q^2$  and  $g(x)$  is a known kinematic function [30]. Taking  $\tilde{m}_Q = 100 \text{ GeV}$  and  $m_{\tilde{g}} = \tilde{m}_Q$ , which implies  $g(1) = 1$ , we find

$$\frac{|m_{\tilde{Q}_1}^2 - m_{\tilde{Q}_3}^2|}{(2m_{\tilde{Q}_1} + m_{\tilde{Q}_3})^2} < 0.45 \left( \frac{\tilde{m}_Q}{100 \text{ GeV}} \right)^2. \quad (57)$$

It should be mentioned that, by carefully tuning the squark and gluino masses, one finds a “sweet spot” in parameter space, where an even weaker bound is obtained [31].

## 7 Warped Extra Dimension

The next framework that we analyze is the Randall-Sundrum (RS) warped extra dimension [32]. Here we consider a generic form of this theory, in which all matter fields propagate in the bulk, while the Higgs field is confined to the IR brane. The localization of the fermions in the bulk generates mass hierarchies and mixing angles, thus addressing the flavor puzzle [33, 34, 35, 36]. Moreover, there is an inherent mechanism in this framework which provides protection against large FCNCs, namely RS-GIM [37].

The  $\Delta F = 2$  operator in Eq. (4) is most dominantly induced by a tree level Kaluza-Klein (KK) gluon exchange. Focusing on this contribution, we can write

$$m_{\text{KK}} = \Lambda_{\text{NP}}, \quad X_Q^{\Delta F=2} \cong \frac{g_{s*}}{\sqrt{6}} \text{diag}(f_{Q^1}^2, f_{Q^2}^2, f_{Q^3}^2), \quad (58)$$

before removing the trace, where  $g_{s*}$  is the dimensionless 5D coupling of the gluon ( $g_{s*} \approx 3$  after one loop matching [38]) and the  $f_{Q^i}$ ’s are the values of the quark doublets on the IR brane. These

are related to each other through the CKM elements  $-f_{Q^1,Q^2}/f_{Q^3} \sim V_{ub}, V_{cb}$ . Plugging the length of  $X_Q^{\Delta F=2}$  calculated from Eq. (58) into Eq. (39), the resulting limit is

$$m_{\text{KK}} > 0.4 f_{Q^3}^2 \text{ TeV}, \quad (59)$$

where  $f_{Q^3}$  is typically in the range of  $0.4\text{--}\sqrt{2}$ .

The  $\Delta F = 1$  operator in Eq. (23) is generated, among others, via mixing between the zero mode (SM) Z boson and its KK excitations. The bulk profile of these higher modes is localized near the IR brane, which results in non-universal couplings to the fermions. This in turn generates flavor violating couplings in the mass basis [37, 39], roughly given by

$$\delta g_Z \cong \log \left( \frac{M_{\text{Pl}}}{\text{TeV}} \right) \left( \frac{m_Z}{m_{\text{KK}}} \right)^2, \quad (60)$$

where  $M_{\text{Pl}}$  is the Planck scale. We focus only on this contribution, as the others are of the same order [39] (see also [40] for a recent discussion on RS flavor violation in the up sector), and write

$$X_Q^{\Delta F=1} \cong g_{Z*} \delta g_Z \text{diag}(f_{Q^1}^2, f_{Q^2}^2, f_{Q^3}^2), \quad (61)$$

with  $g_{Z*}$  as the dimensionless 5D coupling of the Z to left-handed up type quarks ( $g_{Z*} \cong 1.2$  at one loop). The bound that stems from this via case (ii) of Eq. (32) is

$$m_{\text{KK}} > 0.33 f_{Q^3}^2 \text{ TeV}. \quad (62)$$

The constraints presented in Eqs. (59) and (62) are rather weak, compared to known limits on RS, but they are immune to various models of alignment [41].

## 8 Conclusions & Outlook

The field of flavor physics has arrived to a point where it is clear that flavor and CP violation are dominated by the standard model Cabibbo-Kobayashi-Maskawa mechanism. However, at this time we cannot determine what will be the nature of the expected, yet to be discovered, new dynamics at the TeV scale. Our current indirect experimental data is certainly not mature enough to point to a flavor blind dynamics. However, if the new physics is non-universal, we can, with reasonable certainty, expect that, if accessible to the LHC, it would share the approximate symmetry structure of the SM. Thus flavor should be dominantly broken via the third generation sector [42]. It looks, therefore, useful to derive a flexible TeV effective description for flavor violation, that allows to manifestly incorporate the standard model form of flavor breaking. Our covariant formalism enables us not only to describe generic new physics in a model independent manner, but also to naturally describe the SM breaking pattern, given that the formalism's basic building blocks are the SM sources of flavor breaking.

We find that projected LHC bounds on  $\Delta t = 1$  processes lead to a new model independent constraint on the strength of left-handed quarks flavor violation, even in the presence of general flavor alignment mechanisms. The projected bound on  $\Delta t = 2$  transitions from same sign tops production at the LHC is also studied. In this case we find a stronger bound due to recent experimental constraint on CP violation in  $D - \bar{D}$  mixing. We use our analysis to obtain new limits on supersymmetric and warped extra dimension models of alignment, which turn out to be rather weak, but nonetheless replacing practically non-existing current bounds.

In our study we have only focused on the leading framework independent contributions. We want to point out that in our  $\Delta F = 1$  analysis we have only considered the contribution from the operator  $O_{LL}^h$ , given that  $O_{LL}^u$  induces  $b \rightarrow s$  transitions only at one loop. Nevertheless, it is interesting to analyze the bounds induced by the latter operator. Moreover, the supersymmetric contributions that we have considered above are actually not coming from  $O_{LL}^h$  [28], and in any case were found to be very weak. It is therefore worthwhile to study the role of other contributions mediated by the Higgs and chargino sector, depending on additional parameters. It is also worth mentioning that the set of operators discussed above lead to other flavor violating processes, such as  $b \rightarrow s\nu\bar{\nu}$ ,  $b \rightarrow \mu\bar{\mu}$  etc., which were not analyzed by us. However, given that the constraints coming from top flavor violation were always much weaker, inclusion of other processes would not change the qualitative nature of our results, yet interesting to further investigate. In addition, one could derive a projected bound on our  $\Delta t = 1$  operators by studying the cases where on-shell top and  $Z$  are obtained in the final state (for a recent work along these lines, but which considered different set of operators see *e.g.* [43] and references therein). We expect that the resulting bounds would be weaker [11], yet to the best of knowledge a dedicated study of this matter does not exist at present.

Another interesting framework that can easily be explored using our formalism is the minimal flavor violation (MFV) scenario [44]. In this case the new flavor breaking source is just a function of the SM Yukawa matrices. It is possible to generate large top flavor violation within this framework [1] and also  $b \rightarrow s$  transitions [44]. We expect that the weakest possible bound in this case would be similar to what we derived in the general scenario. The new physics source would just be an appropriate linear combination of  $Y_u Y_u^\dagger$  and  $Y_d Y_d^\dagger$ . This, however, only corresponds to a narrow subclass of MFV models, denoted as linear MFV [1]. In covariant language, the linear MFV limit simply corresponds to cases where the flavor violating sources reside on the  $Y_u Y_u^\dagger - Y_d Y_d^\dagger$  plane. In general, as our covariant basis explicitly demonstrates, an arbitrary function of the Yukawa matrices could produce any kind of flavor and CP violation [8, 9, 10]. Nonetheless, higher powers of the Yukawas approach the  $U(2)$  limit. Consequently, the general MFV case would be approximately characterized by new physics sources belonging to the submanifold generated in this limit, described in detail in the text. The non-linear limit of MFV is typically obtained in models with large anomalous dimensions or large logs, where third generation Yukawa resummation is required to obtain the low energy effective theory [1, 45]. Thus, distinguishing between these two limiting cases of MFV models could yield precious information on microscopic type of new physics, well beyond the reach of the LHC.

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## A The Number of Different Adjoints in a $\Delta F = 2$ Operator

The analysis above for  $\Delta F = 2$  operators is based on the assumption that these are all written as in Eq. (4). However, one might wonder whether it is possible to consider this form with two

different adjoints, that is  $O_1 \sim (\bar{Q}_i(X_Q)_{ij}\gamma_\mu Q_j) (\bar{Q}_i(Y_Q)_{ij}\gamma^\mu Q_j)$  with  $X_Q \neq Y_Q$ . This means that the new physics is described by two objects, which complicates the analysis significantly.

To address this issue, we appeal to group theory. For three generations, the two quarks in the operator form together a **6** representation of the  $SU(3)_Q$  flavor group, and similarly the two anti-quarks are in a  $\bar{\mathbf{6}}$ . The entire operator together is then part of the reducible representation  $\mathbf{6} \otimes \bar{\mathbf{6}}$ , which decomposes to  $\mathbf{1} \oplus \mathbf{8} \oplus \mathbf{27}$ . Clearly, the operator is therefore described by the symmetric **27** irreducible representation.

Since we are interested in quark structures such as  $\bar{t}c\bar{c}t$  etc., the state inside the **27** representation is of maximal weight. Accordingly, this operator is uniquely defined with a single adjoint object. In other words, if we use two different adjoints,  $X_Q$  and  $Y_Q$ , as suggested above, then only specific linear combination of their components would appear in physical observables, such that they can be absorbed into a single adjoint. This argument of course applies also to the two generations case, for which the operator resides in the **5** (spin 2) representation of  $SU(2)$ .

## B Complete Covariant Basis - Explicit Decomposition

In sec. 3.2 we construct a complete basis for three generations flavor space. In order to give a sense of the physical interpretation of the different basis elements, we present here their decomposition in terms of Gell-Mann matrices, in the down mass basis (writing only the dependence of the leading terms on  $\lambda_C$  and omitting for simplicity  $\mathcal{O}(1)$  factors such as the Wolfenstein parameter  $A$ ). First we show the simpler case of taking the CKM phase to zero, which yields

$$\begin{aligned}
\hat{D}_1 &\sim \{-1, 0, 0, 0, 0, 0, 0, 0\} , \\
\hat{D}_2 &\sim \{0, -1, 0, 0, 0, 0, 0, 0\} , \\
\hat{C}_d^n &\sim \{2\lambda_C, 0, 1, 0, 0, 0, 0, 0\} , \\
\hat{D}_4 &\sim \{0, 0, 0, -1, 0, -\lambda_C, 0, 0\} , \\
\hat{D}_5 &\sim \{0, 0, 0, 0, -1, 0, -\lambda_C, 0\} , \\
\hat{J}_d^n &\sim \{0, 0, 0, -\lambda_C, 0, 1, 0, 0\} , \\
\hat{J}^n &\sim \{0, 0, 0, 0, -\lambda_C, 0, 1, 0\} , \\
\hat{\mathcal{A}}_d^n &= \{0, 0, 0, 0, 0, 0, 0, -1\} ,
\end{aligned} \tag{63}$$

where the values in each set of curly brackets stand for the  $\Lambda_1, \dots, \Lambda_8$  components. This shows which part of an object each basis element extracts under a dot product, relative to the down sector. For instance,  $\hat{D}_1$  is proportional to  $\Lambda_1$ , and therefore represents the real part of a  $2 \rightarrow 1$

transition. Restoring the CKM phase, we find

$$\begin{aligned}
\hat{D}_1 &\sim \{-1, \eta, 0, 0, 0, 0, 0, 0\} , \\
\hat{D}_2 &\sim \{-\eta, -1, 0, 0, 0, 0, 0, 0\} , \\
\hat{C}_d^n &\sim \{2\lambda_C, -2\eta\lambda_C, 1, 0, 0, 0, 0, 0\} , \\
\hat{D}_4 &\sim \{0, 0, 0, -1, \eta, -\lambda_C, -\eta\lambda_C^3, 0\} , \\
\hat{D}_5 &\sim \{0, 0, 0, -\eta, -1, \eta\lambda_C^3, -\lambda_C, 0\} , \\
\hat{J}_d^n &\sim \{0, 0, 0, -\lambda_C, \eta\lambda_C, 1, \eta\lambda_C^2, 0\} , \\
\hat{J}^n &\sim \{0, 0, 0, -\eta\lambda_C, -\lambda_C, -\eta\lambda_C^2, 1, 0\} , \\
\hat{\mathcal{A}}_d^n &= \{0, 0, 0, 0, 0, 0, 0, -1\} ,
\end{aligned} \tag{64}$$

where  $\eta$  is the CPV Wolfenstein parameter. Finally, the leading term decomposition of  $\mathcal{A}_u$  in the down mass basis is

$$\mathcal{A}_u \sim \left\{ -\lambda_C y_c^2 - \lambda_C^5 y_t^2, \eta \lambda_C^5 y_t^2, -(y_c^2 + \lambda_C^4 y_t^2)/2, \lambda_C^3 y_t^2, -\eta \lambda_C^3 y_t^2, -\lambda_C^2 y_t^2, -\eta \lambda_C^4 y_t^2, -y_t^2/\sqrt{3} \right\} , \tag{65}$$

neglecting the mass of the up quark. It is interesting to notice the differences from Eq. (22), where  $\mathcal{A}_u$  is decomposed in the covariant basis. For instance, it is well known that CPV in  $s \rightarrow d$  transitions within the SM is produced only through mixing with the third generation, hence  $\mathcal{A}_u \cdot \Lambda_2$  is suppressed by  $\eta \lambda_C^5 y_t^2$  in Eq. (65). This is in contrast to  $\mathcal{A}_u \cdot \hat{D}_2 \sim \lambda_C y_c^2$  in Eq. (22).

## C Bounds from the First Two Generations

The three generations framework presented in Sec. 3 is oriented at evaluating flavor violating interactions of the bottom and the top. This is manifest from the approximation of neglecting the masses of the first two generation quarks. However, any new physics contribution need not respect this symmetry in general, thus it may lead to flavor violation between the first two generations.

An interesting example for this is the case of  $X_Q^{\Delta F=1} = L \hat{J}_Q$ , mentioned in Sec. 4, which represents full alignment for both sectors regarding third generation decays. Yet it does induce flavor violation between the first two generations, so we can use experimental limits to constrain this form of  $X_Q^{\Delta F=1}$ .

First note that  $\hat{J}_Q$  does not commute with  $\mathcal{A}_d$  and  $\mathcal{A}_u$  anymore, once all quark masses are restored. Thus  $X_Q^{\Delta F=1}$  can maintain this form (written explicitly as  $L(\sqrt{3}\Lambda_3 + \Lambda_8)/2$ ) only in a specific basis. For simplicity, we take it to be in the down mass basis, such that it only induces flavor violation in the up sector, where the constraints are weaker (we avoid tweaking the alignment between the sectors to obtain the weakest bound, since it would only slightly change the result below). Therefore, we need to consider the contribution of  $X_Q^{\Delta F=1}$  to  $c \rightarrow u$  decays. This can be calculated using the complete covariant basis defined in Sec. 3. To do this, we need to construct that basis around the up sector (by replacing  $\mathcal{A}_d \leftrightarrow \mathcal{A}_u$  in the entire derivation). The coefficient of the operator in Eq. (23) is thus given by

$$X_Q^{\Delta F=1} = L \hat{J}_Q \longrightarrow (C_{LL}^h)_{c \rightarrow u} = \left| \left( X_Q^{\Delta F=1} \cdot \hat{D}_1 \right)^2 + \left( X_Q^{\Delta F=1} \cdot \hat{D}_2 \right)^2 \right| = 0.38 L . \tag{66}$$

For the experimental constraint, we take the result given in Eq. (28) of Ref. [46], written in terms of the standard Wilson coefficients. Using the relation between  $C_{LL}^h$  and the Wilson

coefficients [11], we find

$$(C_{LL}^h)_{c \rightarrow u} < 1.05 \left( \frac{\Lambda_{\text{NP}}}{1 \text{ TeV}} \right)^2. \quad (67)$$

Plugging Eq. (66) into (67) results in

$$L < 0.59 \left( \frac{\Lambda_{\text{NP}}}{1 \text{ TeV}} \right)^2; \quad \Lambda_{\text{NP}} > 1.7 \text{ TeV}. \quad (68)$$

The above example represents a leading order contribution of new physics to the  $2 \rightarrow 1$  transition. At the end of Sec. 4 we consider a form of  $X_Q^{\Delta F=1}$  designed to avoid such contributions, yet it still generates this process at higher order in  $\lambda_C$ . For instance, the effective CKM matrix as a 2-3 rotation takes  $X_Q^{\Delta F=1} = L \hat{\mathcal{A}}_d$  (complete alignment with the down sector) into a combination of  $\hat{\mathcal{A}}_d$ ,  $\hat{C}_d$  and  $\hat{J}_d$ , none of which involves transitions among the first two generations. However, if we use the full CKM, as required when all the masses are accounted for, then we do get a contribution to  $c \rightarrow u$  at  $\mathcal{O}(\lambda_C^5)$ , given by

$$X_Q^{\Delta F=1} = L \hat{\mathcal{A}}_d \rightarrow (C_{LL}^h)_{c \rightarrow u} = \left| \left( X_Q^{\Delta F=1} \cdot \hat{D}_1 \right)^2 + \left( X_Q^{\Delta F=1} \cdot \hat{D}_2 \right)^2 \right| = 2.8 \times 10^{-4} L. \quad (69)$$

Plugging this into Eq. (67), the resulting bound is  $L < 800$  for  $\Lambda_{\text{NP}} = 1 \text{ TeV}$ . This is more than two orders of magnitude weaker than the limit from top decay (case (i) in Eq. (32)), and the corresponding bound for the optimal alignment (case (ii)) is of the same order. Hence this suppression is enough to make this bound irrelevant, as compared to the one based on the future LHC top measurements.

Finally, it is instructive to see explicitly the contribution of the covariant basis elements to flavor violation in the first two generations. In the down mass basis, say, these elements can be identified as

$$\begin{aligned} \hat{\mathcal{A}}_d &= -\Lambda_8, \quad \hat{J} = \Lambda_7, \quad \hat{J}_d = \Lambda_6, \quad \hat{C}_d = \Lambda_3, \quad \hat{\mathcal{D}} = \Lambda_{1,2,4,5}, \\ \hat{\mathcal{A}}'_d &= \frac{\Lambda_3 - \sqrt{3}\Lambda_8}{2} = \text{diag}(0, -1, 1), \quad \hat{J}_Q = \frac{\sqrt{3}\Lambda_3 + \Lambda_8}{2} = \frac{1}{\sqrt{3}} \text{diag}(2, -1, -1). \end{aligned} \quad (70)$$

Clearly, only  $\Lambda_{1,2}$  generate the transition  $s \rightarrow d$ . When a full CKM rotation is applied to move to the up mass basis, the following contributions to  $c \rightarrow u$  arise:

- The  $\Lambda_{1,2}$  components of  $\hat{\mathcal{D}}$  yield a direct contribution.
- $\hat{C}_d$ ,  $\hat{\mathcal{A}}'_d$  and  $\hat{J}_Q$  all contain  $\Lambda_3$ , thus producing an  $\mathcal{O}(\lambda_C)$  contribution.
- The  $\Lambda_{4,5}$  components of  $\hat{\mathcal{D}}$  generate this contribution via a 2-3 rotation, which is at  $\mathcal{O}(\lambda_C^2)$ .
- $\hat{J}$  and  $\hat{J}_d$  require a 1-3 rotation, of  $\mathcal{O}(\lambda_C^3)$ .
- For  $\hat{\mathcal{A}}_d$ , a combination of a 2-3 and a 1-3 CKM rotation is needed, which is at  $\mathcal{O}(\lambda_C^5)$ .

In Sec. 4 we throw away the  $\hat{\mathcal{D}}$  component of  $X_Q^{\Delta F=1}$  (among others), claiming that it contributes to  $2 \rightarrow 1$  at leading order. Here we see that actually the  $\Lambda_{4,5}$  parts of  $\hat{\mathcal{D}}$  are only relevant at  $\mathcal{O}(\lambda_C^2)$ . Nonetheless, these do contribute to both the top and the bottom decays. Then, since under the CKM transformation they mostly mix with  $\Lambda_{1,2}$ , minimizing their contribution requires large  $\Lambda_{1,2}$  components, which in turn directly produce  $2 \rightarrow 1$ .

# References

- [1] A. L. Kagan, G. Perez, T. Volansky and J. Zupan, Phys. Rev. D **80**, 076002 (2009) [arXiv:0903.1794 [hep-ph]].
- [2] G. Perez, arXiv:0911.2092 [hep-ph].
- [3] M. Schmaltz and D. Tucker-Smith, Ann. Rev. Nucl. Part. Sci. **55**, 229 (2005) [arXiv:hep-ph/0502182].
- [4] O. Gedalia, L. Mannelli and G. Perez, arXiv:1002.0778 [hep-ph].
- [5] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. D **82**, 032001 (2010) [arXiv:1005.2757 [hep-ex]].
- [6] T. Feldmann, M. Jung and T. Mannel, Phys. Rev. D **80**, 033003 (2009) [arXiv:0906.1523 [hep-ph]]. E. E. Jenkins and A. V. Manohar, JHEP **0910**, 094 (2009) [arXiv:0907.4763 [hep-ph]].
- [7] K. Blum, Y. Grossman, Y. Nir and G. Perez, Phys. Rev. Lett. **102**, 211802 (2009) [arXiv:0903.2118 [hep-ph]].
- [8] G. Colangelo, E. Nikolidakis and C. Smith, Eur. Phys. J. C **59**, 75 (2009) [arXiv:0807.0801 [hep-ph]].
- [9] L. Mercolli and C. Smith, Nucl. Phys. B **817**, 1 (2009) [arXiv:0902.1949 [hep-ph]].
- [10] J. Ellis, R. N. Hodgkinson, J. S. Lee and A. Pilaftsis, arXiv:0911.3611 [hep-ph]; J. R. Ellis, J. S. Lee and A. Pilaftsis, Phys. Rev. D **76**, 115011 (2007) [arXiv:0708.2079 [hep-ph]].
- [11] P. J. Fox, Z. Ligeti, M. Papucci, G. Perez and M. D. Schwartz, Phys. Rev. D **78**, 054008 (2008) [arXiv:0704.1482 [hep-ph]].
- [12] W. Buchmuller and D. Wyler, Nucl. Phys. B **268**, 621 (1986); J. A. Aguilar-Saavedra, Nucl. Phys. B **812**, 181 (2009) [arXiv:0811.3842 [hep-ph]]; K. Whisnant, J. M. Yang, B. L. Young and X. Zhang, Phys. Rev. D **56**, 467 (1997) [arXiv:hep-ph/9702305].
- [13] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **93**, 081802 (2004) [arXiv:hep-ex/0404006].
- [14] M. Iwasaki *et al.* [Belle Collaboration], Phys. Rev. D **72**, 092005 (2005) [arXiv:hep-ex/0503044].
- [15] J. Carvalho *et al.* [ATLAS Collaboration], Eur. Phys. J. C **52**, 999 (2007) [arXiv:0712.1127 [hep-ex]].
- [16] F. Larios and F. Penunuri, J. Phys. G **30**, 895 (2004) [arXiv:hep-ph/0311056].
- [17] S. Kraml and A. R. Raklev, Phys. Rev. D **73**, 075002 (2006) [arXiv:hep-ph/0512284].
- [18] J. Gao, C. S. Li, X. Gao and Z. Li, Phys. Rev. D **78**, 096005 (2008) [arXiv:0808.3302 [hep-ph]].
- [19] J. Alwall *et al.*, JHEP **0709**, 028 (2007) [arXiv:0706.2334 [hep-ph]].



- [20] C. Amsler *et al.* [Particle Data Group], Phys. Lett. B **667**, 1 (2008).
- [21] O. Gedalia, Y. Grossman, Y. Nir and G. Perez, Phys. Rev. D **80**, 055024 (2009) [arXiv:0906.1879 [hep-ph]].
- [22] I. I. Bigi, arXiv:0907.2950 [hep-ph]; I. I. Bigi, M. Blanke, A. J. Buras and S. Recksiegel, JHEP **0907**, 097 (2009) [arXiv:0904.1545 [hep-ph]]; A. L. Kagan and M. D. Sokoloff, Phys. Rev. D **80**, 076008 (2009) [arXiv:0907.3917 [hep-ph]]; E. Golowich, J. Hewett, S. Pakvasa and A. A. Petrov, arXiv:0903.2830 [hep-ph]; M. Ciuchini, E. Franco, D. Guadagnoli, V. Lubicz, M. Pierini, V. Porretti and L. Silvestrini, Phys. Lett. B **655**, 162 (2007) [arXiv:hep-ph/0703204]; A. A. Petrov, arXiv:1003.0906 [hep-ph].
- [23] Y. Nir and N. Seiberg, Phys. Lett. B **309**, 337 (1993) [arXiv:hep-ph/9304307].
- [24] W. Altmannshofer, A. J. Buras and P. Paradisi, arXiv:1001.3835 [hep-ph].
- [25] G. M. de Divitiis, R. Petronzio and L. Silvestrini, Nucl. Phys. B **504**, 45 (1997) [arXiv:hep-ph/9704244].
- [26] C. S. Li, R. J. Oakes and J. M. Yang, Phys. Rev. D **49**, 293 (1994) [Erratum-ibid. D **56**, 3156 (1997)]; G. Couture, C. Hamzaoui and H. Konig, Phys. Rev. D **52**, 1713 (1995) [arXiv:hep-ph/9410230]; J. j. Cao, Z. h. Xiong and J. M. Yang, Nucl. Phys. B **651**, 87 (2003) [arXiv:hep-ph/0208035]; G. Eilam, A. Gemintern, T. Han, J. M. Yang and X. Zhang, Phys. Lett. B **510**, 227 (2001) [arXiv:hep-ph/0102037]; J. Guasch and J. Sola, Nucl. Phys. B **562**, 3 (1999) [arXiv:hep-ph/9906268].
- [27] E. Lunghi, A. Masiero, I. Scimemi and L. Silvestrini, Nucl. Phys. B **568**, 120 (2000) [arXiv:hep-ph/9906286];
- [28] G. Buchalla, G. Hiller and G. Isidori, Phys. Rev. D **63**, 014015 (2000) [arXiv:hep-ph/0006136]; A. Ali, P. Ball, L. T. Handoko and G. Hiller, Phys. Rev. D **61**, 074024 (2000) [arXiv:hep-ph/9910221]. E. Lunghi and I. Scimemi, Nucl. Phys. B **574**, 43 (2000) [arXiv:hep-ph/9912430]; Q. S. Yan, C. S. Huang, W. Liao and S. H. Zhu, Phys. Rev. D **62**, 094023 (2000) [arXiv:hep-ph/0004262]; A. Ali, E. Lunghi, C. Greub and G. Hiller, Phys. Rev. D **66**, 034002 (2002) [arXiv:hep-ph/0112300].
- [29] G. Raz, Phys. Rev. D **66**, 037701 (2002) [arXiv:hep-ph/0205310].
- [30] M. Ciuchini *et al.*, JHEP **9810**, 008 (1998) [arXiv:hep-ph/9808328].
- [31] A. Crivellin and M. Davidkov, arXiv:1002.2653 [hep-ph].
- [32] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999) [arXiv:hep-ph/9905221].
- [33] H. Davoudiasl, J. L. Hewett and T. G. Rizzo, Phys. Lett. B **473**, 43 (2000) [arXiv:hep-ph/9911262].
- [34] A. Pomarol, Phys. Lett. B **486**, 153 (2000) [arXiv:hep-ph/9911294].
- [35] Y. Grossman and M. Neubert, Phys. Lett. B **474**, 361 (2000) [arXiv:hep-ph/9912408].
- [36] T. Gherghetta and A. Pomarol, Nucl. Phys. B **586**, 141 (2000) [arXiv:hep-ph/0003129].

- [37] K. Agashe, G. Perez and A. Soni, Phys. Rev. D **71**, 016002 (2005) [arXiv:hep-ph/0408134].
- [38] K. Agashe, A. Azatov and L. Zhu, Phys. Rev. D **79**, 056006 (2009) [arXiv:0810.1016 [hep-ph]].
- [39] K. Agashe, G. Perez and A. Soni, Phys. Rev. D **75**, 015002 (2007) [arXiv:hep-ph/0606293].
- [40] M. Bauer, S. Casagrande, U. Haisch and M. Neubert, arXiv:0912.1625 [hep-ph].
- [41] C. Csaki, G. Perez, Z. Surujon and A. Weiler, arXiv:0907.0474 [hep-ph]; C. Csaki, A. Falkowski and A. Weiler, Phys. Rev. D **80**, 016001 (2009) [arXiv:0806.3757 [hep-ph]]; J. Santiago, JHEP **0812**, 046 (2008) [arXiv:0806.1230 [hep-ph]]; A. L. Fitzpatrick, L. Randall and G. Perez, Phys. Rev. Lett. **100**, 171604 (2008) [arXiv:0710.1869 [hep-ph]]; G. Perez and L. Randall, JHEP **0901**, 077 (2009) [arXiv:0805.4652 [hep-ph]].
- [42] Z. Ligeti, M. Papucci and G. Perez, Phys. Rev. Lett. **97**, 101801 (2006) [arXiv:hep-ph/0604112]; K. Agashe, M. Papucci, G. Perez and D. Pirjol, arXiv:hep-ph/0509117.
- [43] J. A. Aguilar-Saavedra, arXiv:1003.3173 [hep-ph].
- [44] G. D'Ambrosio, G. F. Giudice, G. Isidori and A. Strumia, Nucl. Phys. B **645**, 155 (2002) [arXiv:hep-ph/0207036]; T. Hurth, G. Isidori, J. F. Kamenik and F. Mescia, Nucl. Phys. B **808**, 326 (2009) [arXiv:0807.5039 [hep-ph]]; A. J. Buras, Acta Phys. Polon. B **34**, 5615 (2003) [arXiv:hep-ph/0310208]; A. J. Buras, arXiv:hep-ph/0505175; G. Isidori, arXiv:0908.0404 [hep-ph]; L. J. Hall and L. Randall, Phys. Rev. Lett. **65**, 2939 (1990); R. S. Chivukula and H. Georgi, Phys. Lett. B **188**, 99 (1987); A. J. Buras, P. Gambino, M. Gorbahn, S. Jager and L. Silvestrini, Phys. Lett. B **500**, 161 (2001) [arXiv:hep-ph/0007085]; A. Ali and D. London, Eur. Phys. J. C **9**, 687 (1999) [arXiv:hep-ph/9903535]; E. Gabrielli and G. F. Giudice, Nucl. Phys. B **433**, 3 (1995) [Erratum-ibid. B **507**, 549 (1997)] [arXiv:hep-lat/9407029].
- [45] G. Isidori, Y. Nir and G. Perez, arXiv:1002.0900 [hep-ph].
- [46] S. Fajfer, N. Kosnik and S. Prelovsek, Phys. Rev. D **76**, 074010 (2007) [arXiv:0706.1133 [hep-ph]].